Discrete Mathematics Curriculum - Overview

This course is designed for students who have completed their regular mathematics courses through Algebra II (and possibly Trigonometry and Advanced Math) and who do not have either a specific need or an interest in taking Calculus.

The content of Discrete Math's includes the mathematics of making social decisions, management methodology, analysis of data, basic statistics, basic right triangle trigonometry, making projections of future trends, basic probability and financial decision-making. In line with its objectives, the approach of this course will be problem solving and applications, with students encouraged to make conjectures about methods of solution. This course will make students aware of a variety of techniques for approaching and solving real-world problems; students will also develop the ability to apply these techniques to new problems. Furthermore, group work will be utilized to develop students' ability to work with others. Finally, students should acquire a sense of the utility and value of mathematics beyond the classroom.

Discrete Mathematics Curriculum Mapping - Standards

Unit 1	Unit 2	Unit 3	Unit 4
S.CP.A.1	A.CED.1	A.CED.1	A.CED.1
S.CP.A.4	S.CP.A.1	A.CED.2	S.MD.B.7
N.VMA.3	S.CP.A.2		
	S.CP.A.3		
	S.CP.A.4		
	S.CP.A.5		
	S.CP.B.6		
	S.CP.B.7		
	S.CP.B.8		
	S.CP.B.9		
	S.MD.A.2		
	S.MD.A.3		

PCTI MATHEMATICS DEPARTMENT		
Discrete Mathematics	**UNIT 1**	**Logic, Urban Services, & Business Efficiency**

TECHNOLOGY STANDARDS	KEY VOCABULARY		
• Symbolic logic overview http://www.youtube.com/watch?v=OLGVhszBlq4 • Statements, truth values and truth tables http://www.math.csusb.edu/notes/logic/lognot/node1.html http://www.math.csusb.edu/notes/quizzes/tablequiz/tablepractice.html • Logical equivalence and implication http://www.math.csusb.edu/notes/logic/lognot/node2.html • • Euler paths and circuits http://www.youtube.com/watch?v=5M-m62qTR-s • Hamiltonian Circuits http://www.youtube.com/watch?v=AamHZhAmR7o http://www.youtube.com/watch?v=uFCq7e4QynI	Statement Negation Converse Inverse Contrapositive Truth Value Truth table Equivalent Implies If-Then If and Only If (IFF)	Circuit Chinese postman problems Connected Graph Digraph Edge Euler circuit Eulerizing Graph Operations Research Optimal Solution Path Valence Vertex	Algorithm Brute Force Complete graph Critical Path Fundamental Principal of Counting Greedy Algorithm Hamiltonian circuit Kruskal's Algorithm Minimum-cost Nearest neighbor Order-requirement digraph Sorted-edges Spanning tree Traveling salesman problem (TSP) Tree Weight

#	TOPICS (textbook reference; # of days for instruction)	#	STUDENT LEARNING OBJECTIVES	CCSS code
I	**Logic** **(15 days)**			
	1day	**1**	Understand Statements	S.CP.1
	1day	**2**	Determine negations of statements	S.CP.1
	1day	**3**	Determine converse of statements	S.CP.1
	1day	**4**	Determine inverse of statements	S.CP.1
	2days	**5**	Determine contrapositive of statements	S.CP.1
	3days	**6**	Create Truth tables	S.CP.4
	2days	**7**	Determine equivalence statements	S.CP.4
	2days	**8**	Determine if p implies q	S.CP.4
II	**Urban Services** **(1.1-1.4 ; 10 days)**			
	2days	**1**	Determine if graphs have Euler Circuits	N.VMA.3
	2days	**2**	Find Euler Circuits	N.VMA.3
	2days	**3**	Eulerize Graphs	N.VMA.3
	2days	**4**	Solve Urban graph transversal prblems	N.VMA.3
III	**Business Efficiency** **(2.1-2.5 ; 15 days)**			
	1day	**1**	Determine if a graph has a Hamiltonian circuit	N.VMA.3
	2days	**2**	Solve traveling salesman problems	N.VMA.3
	4days	**3**	Use algorithms to solve TSP	N.VMA.3
	3days	**4**	Identify minimal cost spanning trees	N.VMA.3
	3days	**5**	Identify and analyze critical paths	N.VMA.3

Selected Opportunities for Connections to Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Code #	Common Core State Standards
S.CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
S.CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
N.VMA.3	Solve problems involving velocity and other quantities that can be represented by vectors.

PCTI MATHEMATICS DEPARTMENT		
Discrete Mathematics	**UNIT 2**	**Planning & Scheduling, Probability**

TECHNOLOGY STANDARDS	KEY VOCABULARY	
• Explore machine scheduling. https://www.youtube.com/watch?v=PgVpw5JIt7I https://www.youtube.com/watch?v=3dYJRpsmSgY • Bin packing algorithms. https://www.youtube.com/watch?v=vUxhAmfXs2o&list=PLrLvyojgydA786hLKOc0_Jk0_BdhMZeUQ https://www.youtube.com/watch?v=Ll8A5wDwR6I • Graph coloring and conflict resolution. https://www.youtube.com/watch?v=jCuFGCyzdk0 • Explore probability models. https://www.youtube.com/watch?v=fZwKpkhyI_k https://www.youtube.com/watch?v=_vILIwhaqAQ https://www.youtube.com/watch?v=JNm3M9cqWyc https://www.youtube.com/watch?v=se_VxWxiIfc	Average-case analysis Bin-packing problem Chromatic number Critical-path scheduling Decreasing-time-list algorithm First-fit (FF) First-fit decreasing (FFD) Heuristic algorithm Independent task List-processing algorithm Machine scheduling problem Next-fit (NF) Next-fit decreasing (NFD) Priority list Processor Ready task Vertex coloring Worst-case analysis Worst-fit (WF) Worst-fit decreasing (WFD)	Addition rule Central limit theorem Combinatorics Complement rule Continuous probability model Density curve Discrete probability model Disjoint event Event Law of large numbers Mean of a probability model Probability Probability histogram Probability model Random phenomenon Sample space Sampling distribution Standard deviation Variance

#	TOPICS (textbook reference; # days for instruction)	#	STUDENT LEARNING OBJECTIVES	CCSS code
IV	**Planning and Scheduling** **(3.1-3.5 ; 16 days)**			
	5 days	**1**	Schedule task on two and three “machines”	A.CED.1
	2 days	**2**	Create critical paths	A.CED.1
	2 days	**3**	Schedule independent tasks	A.CED.1
	3 days	**4**	Find efficient ways to pack “bins”	A.CED.1
	2 days	**5**	Solve conflicts using colors	A.CED.1
V	**Probability: The Mathematics of Chance** **(8.1-8.6 ; 20 days)**			
	3 days	**1**	Probability models and rules	S.CP.A.1-5
	2 days	**2**	Discrete probability models	CP.B.6-9
	3 days	**3**	Equally likely events	CP.A.2
	3 days	**4**	Continuous probability models	MD.A.3
	3 days	**5**	The mean and standard deviation of a probability model	MD.A.2
	3days	**6**	The central limit theorem	MD.A.2

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8. Look for and express regularity in repeated reasoning.

Code #	Common Core State Standards
A.CED.1	Create equations and inequalities in one variable and use them to solve problems.
S.CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
S.CP.A.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization determine if they are independent.to
S.CP.A.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
S.CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
S.CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations
S.CP.B.6	Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
S.CP.B.7	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.
S.CP.B.8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
S.CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
S.MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
S.MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

PCTI MATHEMATICS DEPARTMENT		
Discrete Mathematics	**UNIT 3**	**Social Choice, Weighted Voting Systems, Electing the President**

TECHNOLOGY STANDARDS		**KEY VOCABULARY**	
• Explore methods of social choice. https://www.youtube.com/watch?v=5YgRDV_hEMc https://www.youtube.com/watch?v=5RtOCvFqIKk https://www.youtube.com/watch?v=FdWMMQlNIt4 • Explore weighted voting systems. https://www.youtube.com/watch?v=5QBxgkpe8ks https://www.youtube.com/watch?v=6T7g4AyMIm0 https://www.youtube.com/watch?v=sdWgGzetdWI • Explore how the president is elected. https://www.youtube.com/watch?v=OUS9mM8Xbbw https://www.youtube.com/watch?v=7wC42HgLA4k	Agenda Approval Voting Arrow's impossibility theorem Borda count Condorcet's method Condorcet winner Condorcet winner criterion Condorcet winner paradox Hare system Independence of irrelevant alternatives Manipulability Majority rule May's theorem Monotonicity Pareto condition Plurality runoff Plurality voting Preference list ballot Sequential pairwise voting	Addition formula Banzhaf power index Bit Binary number Blocking Coalition Coalition Critical voter Dictator Duality formula Dummy Equivalent voting systems Extra votes Extra-votes principle Factorial Losing coalition Minimal winning coalition Pascal's triangle Pivotal voter Power index Quota Shapley-Shubik power index Veto power Voting combination	Bandwagon effect Dichotomous preference Discrete distribution of voter Dominant strategy Electoral College Equilibrium position Expected electoral vote Expected popular vote Extended median Global minimum Local maximum Maximin position Median-voter position 1/3-seperation obstacle Poll assumption Proportional rule Sincere voting Spatial models Spoiler problem Strategic voting 3/2's rule 2/3-seperation opportunity Voter distribution

#	TOPICS (textbook reference; # of days for instruction)	#	STUDENT LEARNING OBJECTIVES	CCSS code
VI	**Social Choice: The Impossible Dream (9.1-9.4 ; 14 days)**			
	5 days	**1**	Understanding Majority Rule and Condorcet's method	A.CED.1
	2 days	**2**	Discover other voting systems for three or more candidates	A.CED.2
	2 days	**3**	Insurmountable difficulties: Arrows impossibility Theorem	A.CED.2
	3 days	**4**	A better approach? Approval voting	A.CED.1
VII	**Weighted Voting Systems (11.1-11.3; 11 days)**			
	3 days	**1**	Understand the Shapley-Shubik Power Index	A.CED.1
	3 days	**2**	Understand the Banzhaf Power index	A.CED.1
	3 days	**3**	Compare voting systems	A.CED.1
VIII	**Electing the President (12.1 – 12.7; 16 days)**			
	2 days	**1**	Spatial models for two candidate systems	A.CED.1
	2 days	**2**	Spatial models for multicandidate elections	A.CED.1
	2 days	**3**	Winnowing the field	A.CED.1
	2 days	**4**	What drives candidates out	A.CED.1
	2 days	**5**	Election Reform: approval voting	A.CED.1
	2 days	**6**	The electoral college	A.CED.1
	2 days	**7**	Is there a better way to elect a President	

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Code #	Common Core State Standards
A.CED.1	Create equations and inequalities in one variable and use them to solve problems.
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

PCTI MATHEMATICS DEPARTMENT		
Discrete Mathematics	**UNIT 4**	**Fair division, Apportionment, Game theory: The Mathematics of Competition**

TECHNOLOGY STANDARDS	KEY VOCABULARY		
• Explore the methods of Fair Division https://www.youtube.com/results?search_query=fair+division+methods https://www.youtube.com/watch?v=RkRWwFhSlog https://www.youtube.com/watch?v=i3YdEGmnJd0 https://www.youtube.com/watch?v=xOZUKsA8pOo • Explore the methods of apportionment. https://www.youtube.com/watch?v=aJrHFDlNtyM https://www.youtube.com/watch?v=YWfEqWLz9pc https://www.youtube.com/watch?v=weGGVmy9yLc https://www.youtube.com/watch?v=ZNybGTvz_hQ https://www.youtube.com/watch?v=l74j-auLjZE • Explore Game theory and strategies. https://www.youtube.com/watch?v=wipSWOp_abo https://www.youtube.com/watch?v=cogQphWYqJE	Adjusted winner procedure Bottom-up strategy Cake-division procedure Convention of the Law of the Sea Divide-and-choose Envy-free Equitable Knaster inheritance procedure Last-diminisher method Lone-divisor method Pareto-optimal Point ratio Preference lists Proportional Selfridge-Conway envy-free procedure Taking turns	Absolute difference Adjusted quota Alabama paradox Apportionment problem Apportionment method Critical divisor District population Divisor method Geometric mean Hamilton method Hill-Huntington method Jefferson method Lower quota Population paradox Quota Quota condition Relative differences Representative share Standard divisor Tentative apportionment Upper quota Webster method	Backward induction Chicken Constant-sum game Dominant strategy Dominated strategy Expected value Fair game Game tree Maximum Minimax Minimax theorem/strategy Mixed strategy Nash equilibrium Nonsymmetrical game Ordinal game Partial-conflict game Payoff matrix Prisoners' dilemma Pure Strategy Rational choice Saddlepoint Status-quo paradox Strategy Theory of moves Total-conflict game Value Variable-sum game

				Zero-sum game
#	**TOPICS (textbook reference; # days for instruction)**	**#**	**STUDENT LEARNING OBJECTIVES**	**CCSS code**
IX	**Fair Division (13.1-13.4 ; 11 days)**			
	2 days	**1**	Divide using the Adjusted Winner Procedure	A.CED.A.1
	3 days	**2**	Divide using The Knaster Inheritance Procedure	A.CED.A.1
	2 days	**3**	Divide by Taking turns	A.CED.A.1
	2 days	**4**	Divide-and-choose	A.CED.A.1
	2 days	**5**		
X	**Apportionment (14.1-14.4; 16 days)**			
	2 days	**1**	Understand the Apportionment Problem	A.CED.A.1
	2 days	**2**	Apportion using the Hamilton Method	A.CED.A.1
	8 days	**3**	Apportion using Divisor Methods	A.CED.A.1
	2 days	**4**	Which Divisor method is best	A.CED.A.1
XI	**Game Theory: The Mathematics of Competition (15.1 – 15.5; 13 days)**			
	2 days	**1**	Two-person total-conflict games: pure strategies	S.MD.B.7
	3 days	**2**	Two-person total-conflict games: mixed strategies	S.MD.B.7
	2 days	**3**	Partial-conflict game	S.MD.B.7
	2 days	**4**	Larger games	S.MD.B.7
	2 days	**5**	Using game theory	S.MD.B.7

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Code #	Common Core State Standards
A.CED.1	Create equations and inequalities in one variable and use them to solve problems.
S.MD.B7	Analyze decisions and strategies using probability concepts.