



**Calculus III**  
Multivariable Calculus  
**Course #0180**  
**Credits 5**  
**2020**

## I. Course Description:

Multivariable Calculus will extend the principles and techniques of single-variable calculus to higher dimensions. Students will study vector algebra and functions, matrices, curves in space, arc length and curvature, and velocity and acceleration. This course is meant for students who successfully completed two years of calculus courses and passed the AP Calculus BC test. Students must be comfortable using or learning to use new technology. Students will use mathematical software, in problem solving, to allow the solution of more complex problems and provide visualization of the mathematical concepts in three dimensions.

## II. Units:

<b>Content Area:</b>	<b>Multivariable Calculus</b>	<b>Grade(s)</b>	<b>12</b>
<b>Unit Plan Title:</b>	<b>Unit 1- Vector Geometry and Calculus of Vector- Valued Functions</b> Vectors in the Plane Vectors in Three Dimensions Dot Product and the Angle Between Two Vectors The Cross Product Planes in 3-Space A Survey of Quadratic Surfaces Cylindrical and Spherical Coordinates Vector -Valued Functions Calculus of Vector – Valued Functions Arc Length and Speed Curvature		
<b>NJSLS Standard(s) Addressed in this unit</b>			
N-VM A. Represent and model with vector quantities. N-VM A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., $v$ , $ v $ , $\ v\ $ , $v$ ). N-VM A.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N-VM A.3 (+) Solve problems involving velocity and other vector quantities that can be represented by vectors. N-VM B. Perform operations on vectors. N-VM B.4 (+) Add and subtract vectors.			

N-VM B.4.a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

N-VM B.4.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

N-VM B.4.c. Understand vector subtraction  $u-w$  as  $u + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM B.5. (+) Multiply a vector by a scalar.

N.VM B.5.a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .

N-VM B.5.b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ).

### Essential Questions (3-5)

How are vectors and scalars similar and different?

How can vector operations be used to model, solve, and interpret real-world problems?

What are the benefits of abstractly representing objects and forces in action?

How are cross products and determinates related to area and volume?

How do you find the velocity and acceleration of an object whose position is represented by a vector – valued function?

### Anchor Text

**Multivariable Calculus Early Transcendentals**. Authors: Jon Rogawski, Colin Adams, Robert Franzosa, 4<sup>th</sup> Edition  
ISBN-13: 978-1-319-05592-9

### Informational Texts (3-5)

Multivariable Calculus, Ron Larson, Bruce H. Edwards. ISBN-13: 978-1337275378

Calculus for AP, Ron Larson, Paul Battaglia. ISBN-13: 978-1305674912

Multivariable Calculus, James Stewart. ISBN-13: 978-1305266643

Basic Multivariable Calculus, Marsden, Jerrold E., Tromba, Anthony J., Weinstein. ISBN-13: 987-0387979762

### Short Texts (1-3)

Multivariable Calculus by James Stewart ISBN-13: 978-1305266643

### Formative & Summative Assessments

#### Formative Assessment

Instructor's observations of notetaking, and assignments  
Class Participation  
Cooperative learning activities  
Observing citizenship and appropriate social responses  
Instructor's observations of time management skills

#### Summative Assessment

Trimester Post Test  
Final Exam  
Project  
Chapter Test

Trimester Pre-Test Quiz Homework Classwork	
<b>Resources (websites, Canvas, LMS, Google Classroom, documents, etc.)</b>	
TI Nspire Cas Graphing Calculator Canvas <a href="https://www.khanacademy.org/math/multivariable-calculus">https://www.khanacademy.org/math/multivariable-calculus</a> <a href="https://www.khanacademy.org/math/linear-algebra/vectors-and-spaces">https://www.khanacademy.org/math/linear-algebra/vectors-and-spaces</a> <a href="https://www.desmos.com/calculator">https://www.desmos.com/calculator</a> <a href="http://www.state.nj.us/education/cccs/2016/math/standards.pdf">http://www.state.nj.us/education/cccs/2016/math/standards.pdf</a> <a href="http://www.state.nj.us/education/cccs/2014/tech/">http://www.state.nj.us/education/cccs/2014/tech/</a> <a href="https://www.cengagebrain.com/shop">https://www.cengagebrain.com/shop</a> <a href="https://login.cengage.com/cb/">https://login.cengage.com/cb/</a> <a href="http://www.macmillanlearning.com">http://www.macmillanlearning.com</a> <a href="http://www.larsoncalculus.com">http://www.larsoncalculus.com</a>	
<b>Suggested Time Frame:</b>	<b>50 days</b>

<b>Content Area:</b>	<b>Multivariable Calculus</b>	<b>Grade(s)</b>	<b>12</b>
<b>Unit Plan Title:</b>	<b>Unit 2- Differentiation in Several Variables</b> Functions of Two or More Variables Limits and Continuity in Several Variables Partial Derivatives Differentiability and Tangent Planes The Gradient and Directional Derivatives The Chain Rule Optimization in Several Variables Lagrange Multipliers: Optimizing with a Constraint		
<b>NJSLS Standard(s) Addressed in this unit</b>			
G-CO A. Experiment with transformations in the plane G-CO A. 1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along the line, and distance along a circular arc. F-IF B. Interpret functions that arise in applications in terms of the context			

F-IF B. 4. For a function that models a relationship between two quantities, interpret key features of the graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing; decreasing; positive; or negative; relative maximums or minimums; symmetries; end behavior; and periodicity.

F-IF B. 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from the graph.

A-REI C. Solve systems of equations

A-REI C. 8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI C. 9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

### Essential Questions (3-5)

What does the limit tell us about a function with several variables?  
 How do you find the derivative of a function with several variables?  
 What information does the derivative provide for functions with several variables?  
 What is a gradient vector and what meaning does it have?  
 How is the chain rule applied when taking derivatives of functions of two variables?

### Anchor Text

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Multivariable Calculus, James Stewart. ISBN-13: 978-1305266643  
Basic Multivariable Calculus, Marsden, Jerrold E., Tromba, Anthony J., Weinstein. ISBN-13: 987-0387979762

### Short Texts (1-3)

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### Formative & Summative Assessments

#### Formative Assessment

Instructor's observations of notetaking, and assignments  
 Class Participation  
 Cooperative learning activities  
 Observing citizenship and appropriate social responses

#### Summative Assessment

Trimester Post Test  
 Final Exam  
 Project  
 Chapter Test

Instructor's observations of time management skills Trimester Pre-Test Quiz Homework Classwork	
<b>Resources (websites, Canvas, LMS, Google Classroom, documents, etc.)</b>	
TI Nspire Cas Graphing Calculator Canvas <a href="https://www.khanacademy.org/math/multivariable-calculus">https://www.khanacademy.org/math/multivariable-calculus</a> <a href="https://www.desmos.com/calculator">https://www.desmos.com/calculator</a> <a href="http://www.state.nj.us/education/cccs/2016/math/standards.pdf">http://www.state.nj.us/education/cccs/2016/math/standards.pdf</a> <a href="http://www.state.nj.us/education/cccs/2014/tech/">http://www.state.nj.us/education/cccs/2014/tech/</a> <a href="https://www.cengagebrain.com/shop">https://www.cengagebrain.com/shop</a> <a href="https://login.cengage.com/cb/">https://login.cengage.com/cb/</a> <a href="http://www.macmillanlearning.com">http://www.macmillanlearning.com</a> <a href="http://www.larsoncalculus.com">http://www.larsoncalculus.com</a>	
<b>Suggested Time Frame:</b>	<b>50 days</b>

<b>Content Area:</b>	<b>Multivariable Calculus</b>	<b>Grade(s)</b>	<b>12</b>
<b>Unit Plan Title:</b>	<b>Unit 3- Multiple Integration</b> Integration in Two Variables Double Integrals over More General Regions Triple Integrals Integration in Polar, Cylindrical, and Spherical Coordinates Applications of Multiple Integrals Change of Variables		
<b>NJSLS Standard(s) Addressed in this unit</b>			
F-IF B. Interpret functions that arise in applications in terms of the context F-IF B. 4. For a function that models a relationship between two quantities, interpret key features of the graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing; decreasing; positive; or negative; relative maximums or minimums; symmetries; end behavior; and periodicity.			

F-IF B. 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from the graph.

G-MG A. Apply geometric concepts in modeling situations

G-MG A. 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-GMD A. Explain volume formulas and use them to solve problems

G-GMD A. 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

### Essential Questions (3-5)

How do we integrate a function of a general, non-rectangular, region?  
 What information does integration provide for functions with several variables?  
 How can double integrals be used to calculate the surface area of a solid?  
 How do you use polar coordinates to simplify the integration of solids over circular regions?

### Anchor Text

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### Formative & Summative Assessments

#### Formative Assessment

Instructor's observations of notetaking, and assignments  
 Class Participation  
 Cooperative learning activities  
 Observing citizenship and appropriate social responses  
 Instructor's observations of time management skills  
 Trimester Pre-Test  
 Quiz  
 Homework

#### Summative Assessment

Trimester Post Test  
 Final Exam  
 Project  
 Chapter Test

Classwork

**Resources (websites, Canvas, LMS, Google Classroom, documents, etc.)**

TI Nspire Cas Graphing Calculator  
Canvas  
<https://www.khanacademy.org/math/multivariable-calculus>  
<https://www.desmos.com/calculator>  
<http://www.state.nj.us/education/cccs/2016/math/standards.pdf>  
<http://www.state.nj.us/education/cccs/2014/tech/>  
<https://www.cengagebrain.com/shop>  
<https://login.cengage.com/cb/>  
<http://www.macmillanlearning.com>  
<http://www.larsoncalculus.com>

**Suggested Time  
Frame:**

**40 days**

<b>Content Area:</b>	<b>Multivariable Calculus</b>	<b>Grade(s)</b>	<b>12</b>
<b>Unit Plan Title:</b>	<b>Unit 4- Line and Surface Integrals and Fundamental Theorems of Vector Analysis</b> Vector Fields Line Integrals Conservative Vector Fields Parametrized Surfaces and Surface Integrals Surface Integrals of Vector Fields Green's Theorem Stokes' Theorem Divergence Theorem		
<b>NJSLS Standard(s) Addressed in this unit</b>			
N-VM A. Represent and model with vector quantities. N-VM A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., $v$ , $ v $ , $\ v\ $ , $v$ ). N-VM A.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N-VM A.3 (+) Solve problems involving velocity and other vector quantities that can be represented by vectors. N-VM B. Perform operations on vectors. N-VM B.4 (+) Add and subtract vectors.			



N-VM B.4.a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

N-VM B.4.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

N-VM B.4.c. Understand vector subtraction  $u-w$  as  $u + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM B.5. (+) Multiply a vector by a scalar.

N-VM B.5.a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .

N-VM B.5.b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ).

G-MG A. Apply geometric concepts in modeling situations

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-GMD A. Explain volume formulas and use them to solve problems

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

### Essential Questions (3-5)

How is a conservative vector field and a potential function related?  
 What are the applications of the scalar line integrals?  
 How do we find area using Green's Theorem?  
 What is Stokes' Theorem used for?

### Anchor Text

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### Formative & Summative Assessments

Formative Assessment

Summative Assessment

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<b>Suggested Time Frame:</b>	<b>40 days</b>

### III. **Instructional Strategies**

- Lecture
- Graphs and other visuals
- Student investigative activities
- Engaging silently and aloud
- Reading silently and aloud
- Brainstorming
- Listening
- Participating in small and large groups
- Collaborative projects

Answering questions (oral and written)  
 Summarizing  
 Debating  
 Peer teaching  
 Note takings  
 Playing games

### **Differentiated Instruction**

Students will work individually, engage in cooperative learning, and utilize discovery learning on certain activities. Using lectures, the internet, and interactive whiteboards, students will be exposed to various teaching methods to appeal to visual, auditory, and kinesthetic learners.

## **IV. Scope and Sequence**

Key: I – Introduced, D – Developed in Depth, R – Reinforced

<b>Skills/ Concepts to be Learned</b>	11	12
Analyze Vectors in the Plane	DR	DR
Understand Vectors in Three Dimensions	IDR	IDR
Use Properties of the Dot Product and Find the Angle Between Two Vectors	IDR	IDR
Use the Properties of the Cross Product	IDR	IDR
Understand Planes in Three-Space	DR	DR
Use Vector-Valued Functions	DR	DR
Find Arc Length and Speed	DR	DR
Compute Curvature	IDR	IDR
Understand Functions of Two or More Variables	IDR	IDR
Find Limits and Continuity in Several Variables	IDR	IDR
Find Partial Derivatives	IDR	IDR
Understand Differentiability and Use Linear Approximation and Tangent Planes	IDR	IDR
Understand and Find Gradient and Directional Derivatives	IDR	IDR
Use the Chain Rule	DR	DR
Use Optimization in Several Variables	IDR	IDR
Use Lagrange Multipliers	IDR	IDR
Understand Integration in Several Variables	IDR	IDR
Find Double Integrals over More General Regions	IDR	IDR

Find Triple Integrals	IDR	IDR
Use Integration in Polar, Cylindrical, and Spherical Coordinates	IDR	IDR
Use Change of Variables	IDR	IDR
Understand Vector Field	IDR	IDR
Evaluate Line Integrals	IDR	IDR
Understand Conservative Vector Fields	IDR	IDR
Understand Parameterized Surfaces and Find Surface Integrals	IDR	IDR
Find Surface Integrals of Vector Fields	IDR	IDR
Understand Green's Theorem	IDR	IDR
Understand Stokes' Theorem	IDR	IDR
Understand Divergence Theorem	IDR	IDR
Use mathematical software, in problem solving, to allow the solution of more complex problems and provide visualization of the mathematical concepts in three dimensions.	IDR	IDR

## V. Complete List Of course Textbooks, Instructional Resources & Software

Multivariable Calculus Early Transcendentals. 4<sup>th</sup> Edition, Copyright Date: 2019, Authors: Jon Rogawski, Colin Adams, Robert Franzosa, Publisher: W. H. Freeman, ISBN-13: 978-1-319-05592-9

Multivariable Calculus, 11<sup>th</sup> Edition, Copyright Date: 20018, Authors: Ron Larson, Bruce H. Edwards, Publisher: Cengage Learning, ISBN-13: 978-1337275378

Calculus for AP, 1<sup>st</sup> Edition, Copyright Date: 2016, Authors: Ron Larson, Paul Battaglia, Publisher: Cengage Learning, ISBN-13: 978-1305674912

Multivariable Calculus, 8<sup>th</sup> Edition, Copyright Date: 2015, Author: James Stewart, Publisher: Cengage Learning, ISBN- 13: 978-1305266643

Basic Multivariable Calculus, 1<sup>st</sup> Edition, Copyright Date: 2017, Authors: Jerrold E. Marsden, Anthony Tromba, Alan Weinstein, Publisher: Springer, ISBN- 13: 987-0387979762

TI Nspire CAS Calculator

Desmos

Exam View

## VI. Student Handout:

### Multivariable Calculus Course Overview

Multivariable Calculus will extend the principles and techniques of single-variable calculus to higher dimensions. Students will study vector algebra and functions, matrices, curves in space, arc length and curvature, and velocity and acceleration. This course is meant for students who successfully completed two years of calculus courses and passed the AP Calculus BC test. Students must be comfortable using or learning to use new technology. Students will use mathematical software, in problem solving, to allow the solution of more complex problems and provide visualization of the mathematical concepts in three dimensions.

### Proficiencies

- Analyze Vectors in the Plane
- Understand Vectors in Three Dimensions
- Use Properties of the Dot Product and Find the Angle Between Two Vectors
- Use the Properties of the Cross Product
- Understand Planes in Three-Space
- Use Vector-Valued Functions
- Find Arc Length and Speed
- Compute Curvature
- Understand Functions of Two or More Variables
- Find Limits and Continuity in Several Variables
- Find Partial Variables
- Understand Differentiability and Use Linear Approximation and Tangent Planes
- Understand and Find Gradient and Directional Derivatives
- Use the Chain Rule
- Use Optimization in Several Variables
- Use Lagrange Multipliers
- Understand Integration in Several Variables
- Find Double Integrals over More General Regions
- Find Triple Integrals
- Use Integration in Polar, Cylindrical, and Spherical Coordinates
- Use Change of Variables
- Understand Vector Field
- Evaluate Line Integrals
- Understand Conservative Vector Field
- Understand Parameterized Surfaces and Find Surface Integrals
- Find Surface Integrals of Vector Fields
- Understand Green's Theorem

Understand Stokes' Theorem  
Understand Divergence Theorem