PASSAIC COUNTY TECHNICAL INSTITUTE
45 Reinhardt Road
Wayne, NJ

Advance Topics in Mathematics Curriculum
Course # 0129
5 Credits
2018

I. Course Description

This is our most advanced course. This course is primarily for students who have completed Calculus BC. The course material is an introduction to the three cornerstones of mathematics, namely, Multivariable Calculus, Differential Equations, and Linear Algebra. The course begins with Multivariable Calculus which involves the application of single-variable calculus concepts to vector functions as well as expand single-variable calculus concepts to functions of more than one variable. Then we cover Differential Equations which consist of methods for solving ordinary differential equations which are studied together with physical applications, numerical solutions, series solutions and laplace transforms. Finally, we discuss the notions of matrices, determinants, systems of linear equations, eigenvalues, and eigenvectors. Each topic will be taught within a trimester. There is a final exam per trimester in addition to a cumulative exam at the end of the year.

II. PCTI Curriculum Unit Planner

Unit 1

Content Area:	**Advance Topics in Mathematics**	**Grade(s)**	**12**
Unit Plan Title:	**Unit 1 – Multivariable Calculus** *Multivariable involve solving functions of two or more variables.* **I.** Functions of Several Variables (32 days) 1. Introduction to functions of Several Variables 2. Limits and Continuity 3. Partial Derivatives 4. Differentials 5. Chain Rules for Functions of Several Variables 6. Directional Derivatives and Gradients 7. Tangent Planes and Normal Lines 8. Extrema of Functions of Two Variables 9. Applications of Extrema		

10. Lagrange Multipliers

II. Multiple Integration (28 days)

1. Iterated Integrals and Area in the Plane
2. Double Integrals and Volumes
3. Change of Variables: Polar Coordinates
4. Center of Mass and Moments of Inertia
5. Surface Area
6. Triple Integrals and Applications
7. Triple Integrals in Other Cordinates
8. Change of Variables: Jacobians

NJSLS Standard(s) Addressed in this unit

G-CO A. Experiment with transformations in the plane
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
FI.F B. Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
G-MG A. Apply geometric concepts in modeling situations
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
G-GMD A. Explain volume formulas and use them to solve problems
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
A-REI C. Solve systems of equations
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Essential Questions (3-5)

1. What does the limit tells us about a function with several variables?
2. How do you find the derivative a functions with several variables?
3. What information does the derivative provide for functions with several variables?

4. How do you reverse differentiation functions with several variables?
5. What information does integration provide for functions with several variables?

Anchor Text

Multivariable Calculus Authors : Ron Larson, Bruce Edwards, 11th Edition,
ISBN 13: 978-1337275378

Informational Texts (3-5)

Ron, Larson, Calculus for AP, Ron Larson, Paul Battaglia
Stewart, James. Multivariable Calculus
Marsden, Jerrold E., Tromba, Anthony J., Weinstein., Basic Multivariable Calculus
Rogawski, Jon, Adams Colin., Calculus Multivariable

Short Texts (1-3)

Multivariable Calculus by James Stewart **ISBN-**0495383635

Formative & Summative Assessments

Formative Assessment

- Instructor's observations of note-taking, and organization of notebooks and assignments
- Class Participation
- Cooperative learning activities
- Observing citizenship and appropriate social responses
- Instructor's observations of time management skills
- Trimester Pre Test

Summative Assessment

- Homework
- Trimester Post Test
- Project
- Final Exam
- Quiz
- Chapter Test

Resources (websites, Canvas, LMS, Google Classroom, documents, etc.)

TI Nspire Cas Graphing Calculator
Canvas
https://www.khanacademy.org/math/multivariable-calculus
https://www.khanacademy.org/math/linear-algebra/vectors-and-spaces

https://www.desmos.com/calculator **http://www.state.nj.us/education/cccs/2016/math/standards.pdf** **http://www.state.nj.us/education/cccs/2014/tech/** **https://www.cengagebrain.com/shop** **https://login.cengage.com/cb/** **http://www.larsoncalculus.com**	
Suggested Time Frame:	60 Days

Unit 2

Content Area:	**Advance Topics in Mathematics**	**Grade(s)**	**12**
Unit Plan Title:	**Unit 2 – Differential Equations** *The laws of nature are expressed as differential equations. Scientists and engineers model the world in terms of differential equations, solve the equations and interpret the solutions. The unit consist of solving linear equations of higher order differential equations, power series and laplace transformation methods.* **I.** Linear Equations of Higher Order (20 days) 1. Introduction: Second-Order Linear Equations 2. General Solutions of Linear Equations 3. Homogeneous Equations with Constant Coefficients 4. Mechanical Vibrations 5. Nonhomogeneous Equations and Undetermined Coefficients **II.** Power Series (16 days) 1. Series Solutions Near Ordinary Points 2. Regular Singular Points 3. Bessel's Equation 4. Applications of Bessel Functions **III.** Laplace Transform Methods (24 days) 1. Laplace Transforms and Inverse Transforms		

2. Transformation of Initial Value Problems
3. Translation and Partial Fractions
4. Derivatives, Integrals, and Products of Transforms
5. Periodic and Piecewise Continuous Input Functions
6. Impulses and Delta Functions

NJSLS Standard(s) Addressed in this unit

A-CED A. Create equations that describe numbers or relationships
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.
A-REI A. Understand solving equations as a process of reasoning and explain the reasoning
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
B. Solve equations and inequalities in one variable
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
C. Solve systems of equations 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
D. Represent and solve equations and inequalities graphically
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Essential Questions (3-5)

1. What is differential equation and how do you solve one?
2. How to recognize and solve homogeneous differential equations?
3. How to use linear differential equations to solve applied problems?
4. How to solve first-order linear equations?

Anchor Text

Elementary Differential Equations and Boundary Value Problems Authors : Boyce, DiPrima and Meade, 11th Edition
ISBN: **978-1119375753**

Informational Texts (3-5)

Ron, Larson, Bruce H. Edwards, Calculus of a Single Variable
Ron, Larson, Calculus for AP, Ron Larson, Paul Battaglia Cengage
Stephen W. Goode, Scott A. Annin. *Differential Equations & Linear Algebra:*
C. Henry Edwards, David E. Penhey, David Calvis. *Differential Equations and Linear Algebra (4th Edition)*

Short Texts (1-3)

Differential Equations for Dummies, 1st Edition by Steven Holzner **ISBN-13:** 978-0470178140
Schaum's Outline of Differential Equations, 4th Edition by Richard Bronson, Gabriel Costa **ISBN-13**: 978-0071824859

Formative & Summative Assessments

Formative Assessment	Summative Assessment
• Instructor's observations of note-taking, and organization of notebooks and assignments • Class Participation • Cooperative learning activities • Observing citizenship and appropriate social responses • Instructor's observations of time management skills • Trimester Pre Test	• Homework • Trimester Post Test • Project • Final Exam • Quiz • Chapter Test

Resources (websites, Canvas, LMS, Google Classroom, documents, etc.)

TI Nspire Cas Graphing Calculator
Canvas
https://www.desmos.com/calculator
https://www.khanacademy.org/math/differential-equations
http://www.state.nj.us/education/cccs/2016/math/standards.pdf
http://www.state.nj.us/education/cccs/2014/tech/
https://www.cengagebrain.com/shop
https://login.cengage.com/cb/

Suggested Time Frame:	60 Days

Unit 3

Content Area:	**Advance Topics in Mathematics**	**Grade(s)**	**12**
Unit Plan Title:	**Unit 3 – Linear Algebra** *Linear algebra is the branch of mathematics concerning linear equations such as linear functions and their representations through matrices and vector spaces.* **I.** Linear Equations in Linear Algebra (18 days) 1. System of Linear Equations 2. Row Reduction and Echelon Form		

3. Vector equations
4. Matrix Equations
5. Solutions of Linear System
6. Linear Independence
7. Linear Transformations
8. Matrix form of Linear Transformations

II. Matrix Algebra (10 days)

1. Matrix Operations
2. Inverse of a Matrix
3. Invertible Matrices
4. LU Factorization

III. Determinants (8 days)

1. Introduction to Determinants
2. Properties of determinants
3. Cramer's Rule

III. Vectors and the Geometry of Space (12 days)

1. Vectors in the plane
2. Space Coordinates and Vectors in Space
3. The Dot product of Two Vectors
4. The Cross Product of Two Vectors in space
5. Lines and Planes in Space

IV. Eigenvalues and Eigenvectors (12 days)

1. Eigenvalues and Eigenvectiors
2. The Characteristic Equations
3. Diagonalization
4. Complex Eigenvalues

NJSLS Standard(s) Addressed in this unit

N-VM C. Perform operations on matrices and use matrices in applications.
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

A-REI C. Solve systems of equations 5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
N-VM A. Represent and model with vector quantities.
1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.
B. Perform operations on vectors.
4. (+) Add and subtract vectors.
a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠0, the direction of cv is either along v (for c > 0) or against v (for c < 0).

Essential Questions (3-5)

1. What are matrices and how do you use them to represent data?
2. How do we find the determinant of a matrix and when is it nonzero?
3. How do we find the inverse of a matrix and when does a matrix not have an inverse defined?
4. How do we solve systems of equations using inverse matrices?
5. What is vectors in a plane?

Anchor Text

Linear Algebra and its Applications Authors : David Lay; Stephen Lay; Judi McDonald, 5th Edition
ISBN: 978-0321982384
Multivariable Calculus Authors : Ron Larson, Bruce Edwards, 11th Edition,
ISBN 13: 978-1-285-06029-3

Informational Texts (3-5)

Stephen W. Goode, Scott A. Annin. *Differential Equations & Linear Algebra:*
C. Henry Edwards, David E. Penhey, David Calvis. *Differential Equations and Linear Algebra (4th Edition)*

Short Texts (1-3)

Linear Algebra for Dummies by Mary Jane Sterling **ISBN-13:** 978-0-43090-3
Linear Algebra Shaums Outline by Seymour Lipschutz and Marc Lars Lipson **ISBN-13:** 978-1260011449

Formative & Summative Assessments

Formative Assessment

- Instructor's observations of note-taking, and organization of notebooks and assignments
- Class Participation
- Cooperative learning activities
- Observing citizenship and appropriate social responses
- Instructor's observations of time management skills
- Trimester Pre-Test

Summative Assessment

- Homework
- Trimester Post Test
- Project
- Final Exam
- Quiz
- Chapter Test

Resources (websites, Canvas, LMS, Google Classroom, documents, etc.)

TI Nspire Cas Graphing Calculator
Canvas
https://www.khanacademy.org/math/linear-algebra
https://www.khanacademy.org/math/precalculus/precalc-matrices
https://www.desmos.com/calculator
http://www.state.nj.us/education/cccs/2016/math/standards.pdf
http://www.state.nj.us/education/cccs/2014/tech/

https://www.cengagebrain.com/shop https://login.cengage.com/cb/	
Suggested Time Frame:	60 Days

III. Instructional Strategies

- Lecture
- Graphs and other visuals
- Student investigative activities
- Engaging in discussions
- Reading silently and aloud
- Brainstorming
- Listening
- Participating in small and large groups
- Collaborative projects
- Answering questions (oral and written)
- Summarizing
- Debating
- Analyzing data, discussions, etc.
- Peer teaching
- Playing games
- Note taking
- Writting

Differentiated Instruction

- Students will work individually, engage in cooperative learning, and utilize discovery learning on certain activities. Through the use of lectures, the internet, and interactive whiteboards, students will be exposed to various teaching methods to appeal to visual, auditory, and kinesthetic learners. Students will be given copies of data sets and other important notes.

IV. Methods of Student Evaluation

Assessment can be divided into two general categories: formal (graded) and informal/classroom-based (both graded and ungraded). The key to effectively assessing a student's mastery of skills is to match the assessment method to the learning objective.

Summative Assessments

- Homework
- Quizzes
- Projects
- Chapter Test
- Final Exam
- Trimester Post Test

Formative Assessments

- Instructor's observations of note-taking, and organization of notebooks and assignments
- Class Participation
- Cooperative learning activities
- Observing citizenship and appropriate social responses
- Instructor's observations of time management skills
- Trimester Pre Test

V. Textbooks, Instructional Resources and Software

1. **Multivariable Calculus,** Authors: Ron Larson and Bruce Edwards, 11th Edition, ISBN 13: 978-1-285-06029-3
2. **Elementary Differential Equations and Boundary Value Problems,** Authors : Boyce and DiPrim, ISBN: 978-0470458310
3. **Linear Algebra and its Applications** Authors : David Lay; Stephen Lay; Judi McDonald, 5th Edition, ISBN: 978-0321982384
4. **TI Nspire Calculator**
5. **Desmos**
6. **Exam View**

VI. Scope and Sequence

Key: I – Introduced, D-developed in Depth, R-Reinforced

Skill/Concepts to be Learned	**11**	**12**
Apply previously developed skills learned in Calculus to learn Multivariable Calculus and Vectors.	IDR	IDR
Recognize and solve homogeneous differential equations.	DR	DR
Solve a first-order linear differential equation	IDR	DR
Solve a second-order linear differential equation		
Use linear differential equations to solve applied problems.	DR	DR
Analyze vectors in space.	DR	DR
Use properties of the dot product of two vectors.	DR	DR
Use triple scalar product of three vectors in space.	DR	DR
Find derivative of a function of two or more variables.	DR	DR
Find integral of a function of two or more variables.	DR	DR
Use double integral to represent the volume of a solid region and use properties of double integrals.	DR	DR
Learn about matrices, determinants, applications to solving linear system of equations, matrix factorization, eigenvalues and eigenvectors.	DR	DR
Use mathematical software, in problem solving, to allow the solution of more complex problems and provide visualization of the same.	DR	DR
Use mathematical software, in problem solving, to allow the solution of more complex problems and provide visualization of the mathematical concepts in three dimensions.	DR	DR

VII. Student Handout

AP Statistics Course Overview

The Advance Topics in Mathematics course is an introduction to Multivariable Calculus. Differential Equations, and Linear Algebra. The course begins with Multivariable Calculus which involves the application of single-variable calculus concepts to vector functions as well as expand single-variable calculus concepts to functions of more than one variable. Then we cover Differential Equations which consist of methods for solving ordinary differential equations which are studied together with physical applications, numerical solutions, series solutions and laplace transforms. Finally, we discuss the notions of matrices, determinants, systems of linear equations, eigenvalues, and eigenvectors. Each topic will be taught within a trimester. There are 3 final tests per trimester including a cumulative exam at the end of the trimesters. Students use technology, investigations, problem solving, and writing as they build conceptual understanding. Students must have taken Calculus BC before enrolling in Advance Topics in Mathematics.

PROFICIENCES
Upon successful completion of the requirements for this course, the student will be able to:
Work with functions of several variables represented in a variety of ways: graphical, analytical, numerical and verbal. They should understand the connections among these representations.
Use derivatives to solve a variety of problems and understand the meaning of the derivative in terms of a rate of change and local linear approximation.
Understand the meaning of the definite integral both as a limit of Riemann Sums and as a net accumulation of a rate of change, and use integrals to solve a variety of problems.
Understand the relationship between derivative and indefinite integral as expressed in Fundamental Theorem of Calculus.
Write and evaluate integrals in polar coordinates.
Discover the higher-order differential equation that describes a specified physical situation, find either exact or approximately the appropriate solution of that equation and nterpret the solution that is found.
Solve linear differential equations with constant coefficients that have numerous applications and can be solved systematically using Laplace transformation.
Use the power series method to solve linear differentitial equation.
Use linear algebra to solve systems of linear equations.
Define three or more dimential space using vectors.
Use eigenvalues and eigenvectors with matrices.
Communicate mathematics both orally and in well-written sentences to explain solutions to a problem.
Model a written description of a physical situation with a function, a differential equation, or an integral.
Use technology to help solve problems, experiment, interpret results and verify conclusions.